Estimate of the Radio Frequency Properties of the Vacuum Chamber

1. Cutoff frequencies

The lowest cutoff frequency is that of the TE waves. A very approximate calculation of the cutoff frequency can be made by considering region II as a capacitance and regions I and III as inductances (see Fig. 1).
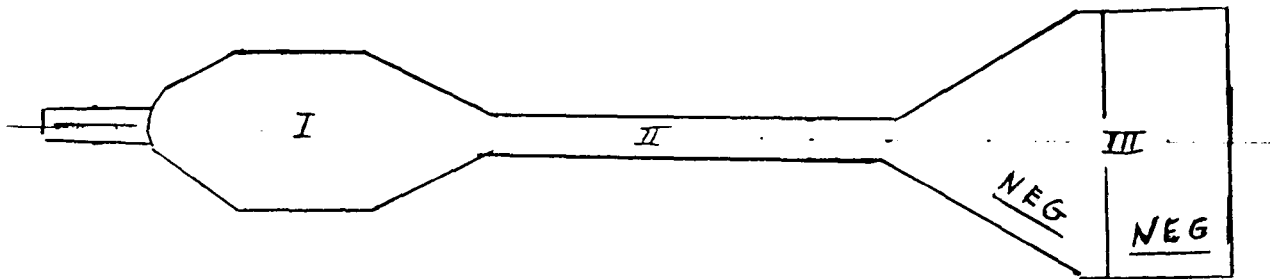

Fig. 1. Vacuum Chamber Cross Section

Region I: beam chamber:

cross section area: $A_I \approx 25$ cm^2

corresponding circumference: $S_I \approx 20$ cm

Region II: gap $g = 1$ cm, width $w = 10$ cm

cutoff frequency: 15 GHz $= 4 \times 10^4 \ f_0$

Region III: cross section area: $A_{III} = 40$ cm^2

corresponding circumference: $S_{III} = 36$ cm

2 NEG strips: each $W_{NEG} = 2$ cm wide

For a length ℓ of the vacuum chamber one has $C_{II} = \dfrac{\varepsilon_0 \ell w}{g}$, $L_I = \dfrac{\mu_0 A_I}{\ell}$,

$L_{III} = \dfrac{\mu_0 A_{III}}{\ell}$. Figure 2 shows the equivalent circuit.
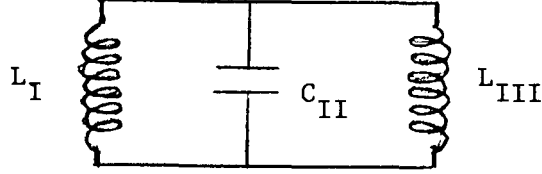

Fig. 2. Equivalent Circuit

The cutoff frequency is given by

$$f_c = \frac{1}{2\pi} \left(\frac{1}{C_{II}} \frac{L_I + L_{III}}{L_I \times L_{III}} \right)^{1/2} = \frac{1}{2\pi} \left[\frac{g}{\mu_0 \varepsilon_0 w} \left(\frac{1}{A_I} + \frac{1}{A_{III}} \right) \right]^{1/2} .$$

Substituting $g = 1$ cm, $w = 10$ cm, $A_I = 25$ cm^2, $A_{III} = 40$ cm^2 and noting that

$\dfrac{1}{\mu_0 \varepsilon_0} = c^2$ (c = light velocity) one obtains

$$f_c \approx 385 \text{ MHz} .$$

The cutoff frequency of region II is 15 GHz so that the cutoff frequency of

the TM waves of region I is mainly determined by its own dimensions

$$f_c \approx \frac{1}{2\pi} \sqrt{\frac{S_I^2}{\mu_0 \varepsilon_0 A_I^2}}$$

$$= \frac{c}{2\pi} \frac{S_I}{A_I} .$$

Setting A_I = 25 cm^2 and S_I = 20 cm, one finds

$$f_c \approx 3.8 \text{ GHz} \ .$$

2. The loss parameter k

Electromagnetic fields are excited by the bunch of positrons passing through the beam chamber. These wake fields may lead to serious problems as bunch lengthening, higher order mode losses etc. for the longitudinal motion and head-tail turbulence etc. for the transverse motion. In this note, the loss parameter k is defined by the relation

$$\Delta U = kq^2,$$

where q is the charge of the bunch and ΔU is the total energy loss per turn. Several components as the rf cavities, chamber shape discontinuities at the insertion device etc. will contribute to the total value of k. In this note only the contribution of the vacuum chamber will be estimated. In this very approximate calculation, the current is assumed to be constant within the bunch length τ and zero elsewhere. Hence the Fourier expansion of the current is given by

$$I = \sum I_n \cos \frac{2\pi n t}{T_0} \ ,$$

where T_0 = rotation period

$$I_n = \frac{2I\tau}{T_0} \frac{\sin \pi n\tau/T_0}{\pi n\tau/T_0}$$

$$= 2I_0 \frac{\sin \pi n\tau/T_0}{\pi n\tau/T_0} \, .$$

Here I_0 = the average circulating current. Replacing the beam chamber by a circular pipe and radius $b = \sqrt{A_I/\pi}$ one obtains for the power dissipated in the wall

$$P = \frac{2\pi R}{2\pi b} I_0^2 R_{SO} \int_0^\infty \frac{\sin^2 \pi \tau n/T_0}{(\pi \tau n/T_0)^2} n^{1/2} \, dn$$

$$= \frac{2\pi R}{2\pi b} \frac{I_0^2}{\pi} (T_0/\tau)^{3/2} R_{SO} \, ,$$

where R_{SO} is the surface resistivity of the vacuum chamber wall at the frequency $f_0 = 1/T_0$. Noting that $q = I_0 T_0$ and $\Delta U = T_0 P$ one finds the loss parameter of the beam chamber (region I)

$$k_I = \frac{1}{\pi} \frac{2\pi R}{2\pi b} \frac{R_{SO}}{T_0} \left(\frac{T_0}{\tau}\right)^{3/2} \, .$$

The cutoff frequency of region II f_N = 15 GHz (g = 1 cm). Thus waves with frequencies above this cutoff value will be transmitted to region III. In this region one has the NEG strips, rough pumping ports etc. In this estimate only the effects of the NEG strips are taken into account. Assuming the dissipated power is proportional to the surface area and the resistivity of the surface material of the wall one obtains for the power loss in region II

$$P_{II} = \frac{2w}{S_I} \frac{2\pi R}{2\pi b} I_0^2 R_{SO} \int_N^\infty \frac{\sin^2 \pi\tau n/T_0}{(\pi\tau n/T_0)^2} n^{1/2} \, dn$$

$$P_{II} \approx \frac{2\pi R}{2\pi b} I_0^2 \left(\frac{T_0}{\pi\tau}\right)^2 \frac{1}{\sqrt{N}} \frac{2w}{S_I} R_{SO} \, , \quad N = f_N/f_0 \, .$$

In the same way one finds for region III

$$P_{III} = \frac{2\pi R}{2\pi b} I_0^2 \left(\frac{T_0}{\pi\tau}\right)^2 \frac{1}{\sqrt{N}} \left[\frac{S_{III}}{S_I} + \frac{2W_{NEG}}{S_I} \frac{\rho_{NEG}}{\rho_{wall}}\right] R_{SO} \, .$$

Substitutions of $\Delta U = T_0 P$ and $q = I_0 T_0$ give

$$k_{II} + k_{III} = \frac{2\pi R}{2\pi b} \frac{1}{T_0} \left(\frac{T_0}{\pi\tau}\right)^2 \frac{1}{\sqrt{N}} \left[\frac{2w + S_{III} + 2W_{NEG} \sqrt{\rho_{NEG}/\rho_{wall}}}{S_I}\right] R_{SO} \, .$$

Setting: $2\pi R = 800$ m, $2\pi b = 0.18$ m, $T_0 = 2.67 \times 10^{-6}$ sec, $N = f_N T_0 = 4 \times 10^4$, $w = 0.1$ m, $S_I = 0.2$ m, $S_{III} = 0.36$ m, $W_{NEG} = 0.02$ m, $\rho_{NEG} = 50 \times 10^{-8}$ ohm-m and $R_{SO} = 2 \times 10^{-4}$ (Aℓ: $\rho_{wall} = 2.7 \times 10^{-8}$ ohm-m, $f_0 = 1/T_0 = 0.375$ MHz), one obtains

$$k_I = 1.5 \times 10^{10} \tau^{-3/2} \text{ Volt/Coulomb}$$

$$k_{II} + k_{III} = 4.4 \times 10^9 \, \tau^{-2} \text{ Volt/Coulomb} \, ,$$

τ in nanoseconds. Note the difference in bunch length dependence.

The relation between the loss parameter and the resistive part of the longitudinal coupling impedance is given by

$$k = \frac{1}{\pi} \int_0^\infty Z_R(\omega) \, e^{-\omega^2 \tau^2} \, d\omega$$

It is customary to approximate the broadband coupling impedance by a parallel R, C, L resonant circuit, having a resonant frequency equal to the vacuum chamber cutoff frequency (ω_c) and a quality factor Q of the order of unity.

$$Z_\parallel(\omega) = \frac{R_0}{1 - jQ\left(\frac{\omega_c}{\omega} - \frac{\omega}{\omega_c}\right)} \quad \text{or}$$

$$Z_R(\omega) = \frac{R_0}{1 + Q^2\left(\frac{\omega_c}{\omega} - \frac{\omega}{\omega_c}\right)^2}$$

Setting $k = k_I + k_{II} + k_{III}$, $Q = 1$ and $\omega_c = 2\pi \times 3.8 \times 10^9$. One obtains for $\tau = 10^{-10}$ sec (3 cm bunch length)

$$R_0 = 3.7 \times 10^3 \text{ ohm}$$

One sees from this result that without including the effects of the pumping holes, valves etc., the coupling impedance of the vacuum chamber is much smaller than the cavities coupling impedance.